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Centre for World Food Studies

**Postponing efficiency: effect on equilibrium selection in economies with
exhaustible resources and amenity values**

by

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Abstract

The paper studies a Ramsey optimal control economy with one consumer good, extended to include an exhaustible resource with amenity value. The steady state equilibria are shown to form a continuum from which the dynamic path selects a unique equilibrium depending on the resource stock left when the efficient policy becomes effective. Hence, postponement of efficient policies has long term effects. If the resource has a very limited, possibly non-convex, regeneration, the continuum becomes a turnpike along which paths move towards the steady state. These results are shown to be robust when the number of consumers and natural resources is increased.

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1. Introduction

While policy makers have increasingly become convinced of the need to implement efficient systems of environmental resource management, experience teaches how difficult it apparently is to put this conviction to practice. Consequently, reforms are being delayed and a host of inefficiencies persist. A sense of urgency might be lacking partly because the economic implications of irreversible degradation of biodiversity loss, soils and climate change, and generally of the long-term effects of postponement are insufficiently understood. Few analysts question the severeness of the physical degradation itself, but if it is possible to compensate the loss by accumulation of man-made capital, the inefficient management of natural resource use does not necessarily cause disaster.

Indeed in the early models on the efficient management of environmental resources, it is always possible to redeem past mistakes [Keeler, Spencer and Zeckhauser 10; Plourde 16]. The effects of postponement also vanish in the classic Ramsey model of optimal growth, with one consumer, one consumer good and concave utility and production functions. The initial conditions of these models can be thought of as reflecting the stock level at the date an efficient resource management policy becomes effective, and neither date nor stock level appear to affect the long run. This is because the solution path converges to a steady state that does not depend on initial conditions. Therefore, this class of models cannot address prevailing environmental concerns which, according to Clark [4], relate to the irreversibility of damages.

This irreversibility is only partly due to the lack of renewability of the resources themselves. Recent papers have argued that it can also result from switching behaviour which is itself due to non-convexity in use and/or regeneration of the resources [Dawid and Kopel 5; Forster 6; Lewis and Schmalensee 12; Majumdar and Mitra 14; Tahvonen and Salo 17]. Most of these studies show within a Ramsey framework, for specified non-convexities and given discount rate, that if the initial resource stock lies below a critical level, the economy can converge to a solution that is distinctly inferior to what is attained when the initial stock exceeds this level. The solution path might also fail to converge, but if the agent's discount rate is sufficiently low it will converge and do so to a steady state that lies close to the golden rule steady state equilibrium. In short, in these models a postponement of efficient policies can have severe consequences for given rate of discount but if the discount rate is sufficiently low, whatever the initial state of the economy, all problems can be overcome.

While this low discount property is generally robust in the papers cited, both the thresholds values and the consequences of crossing them appear to be very much dependent on the assumptions made. We mention three that will be relaxed in this paper. First, most studies only consider the extractive use of the environmental resource, and neglect the service directly rendered by the stock itself, i.e. the amenity value. In this paper we consider an extended Ramsey model with a natural resource that has amenity value. Second, we allow

for an extension in dimensions. Convergence proofs for Ramsey models usually rely on the assumption that there is only one capital good. This is problematic especially in environmental studies because it becomes impossible to combine natural resources and man-made stocks within the same model. Here we initially consider two stocks, one for a natural resource and one for man-made capital. We subsequently allow for an arbitrary number of natural resource stocks, but there will be only one type of man-made capital. Third, the properties of the Ramsey model with non-convexities appear to depend strongly on the type of non-convexity that is being assumed. Interestingly, these models abstract from the basic environmental concern that the scope for regeneration is actually very limited. We show that this makes it possible to incorporate non-convexities of a very general kind, provided the regeneration is sufficiently small.

Our approach will be to characterize the global dynamics of the stocks of the resource and man-made capital through diagrams whose qualitative properties are established in formal propositions, much in the tradition of the turnpike literature from the 1970s [Cass and Shell 3, McKenzie 15]. This literature showed that the long-term behavior of equilibrium paths is closely related to the properties of steady states. For specific finite horizon economies, it was possible to show that if the steady state is unique and stable, the dynamic equilibrium path possesses the turnpike property. It will stay near the steady state for most of the time and only diverge at the beginning and the end. Its initial allocations are insensitive to variations in the far future, and conversely, the effect of variations in initial conditions fades out eventually. In infinite time horizon models such as the Ramsey model, this fading out is referred to as path-independence, and implies that initial conditions have no long-term effects. Though we describe these effects in the specific context of an extended Ramsey model, some of the properties are preserved in more general settings. In any model, the steady states are independent of the initial conditions, precisely because they themselves define the particular initial conditions that allow indefinite replication. Therefore, whenever the steady state exists, and is unique and stable, this fading out will necessarily occur. If steady states are multiple, the situation becomes more complex, and the steady state selected can depend on the initial conditions. It might even happen that no stable steady state is attained from the given initial conditions, and the path could become chaotic [Boldrin and Montrucchio 2].

The paper proceeds as follows. The extended Ramsey model is presented in section 2. The dynamic properties of its basic version, for which the natural resource cannot regenerate, are analyzed in section 3. The model is seen to preserve the global convergence of the classic Ramsey model, but it allows for multiplicity, even for an infinity, of steady state equilibria. Every optimal dynamic path starting from given initial conditions can select a unique element from the set of equilibrium. Hence, path-dependence may occur. Next, in section 4, we consider limited resource regeneration. The dynamic path will stay close to the earlier trajectory, and only bend off at the very end, when it will start moving alongside the continuum, towards the unique steady state. Section 5 considers various generalizations, up to the case of an arbitrary number of natural resources with non-convex technologies, and Section 6 concludes. Proofs are given in the Annex.

2. Model specification

This section extends the classic Ramsey model in continuous time by incorporating a natural resource with amenity value in addition to man-made capital. There is one infinitely-lived consumer. Commodities are a man-made consumer good, man-made capital, and a renewable/exhaustible resource. Time is denoted by a subscript, $t \in [0, \infty)$, which is omitted when convenient. Let k_t be the (renewable) man-made capital stock, let s_t be the (renewable/exhaustible) environmental resource stock, with regeneration function $g(s_t)$ and extraction e_t . Finally, the man-made commodity produced via the production function $f(k_t, e_t)$, serves as consumption, c_t as well as net investments, \dot{k} . The environmental resources have amenity value, i.e. provide a service, and utility is assumed to be separable in time, $u(c_t, s_t)$, and time preference ρ is taken to be constant. The intertemporal welfare program reads

$$\max \int_0^{\infty} e^{-\rho t} u(c, s) dt \quad (1)$$

$$c, k, s, e \geq 0$$

subject to

$$c + \dot{k} \leq f(k, e), \quad (p) \quad (2)$$

$$\dot{s} \leq g(s) - e, \quad (q) \quad (3)$$

for given strictly positive k_0 and s_0 .

Regarding slope and curvature of the functions, we assume that $u(\cdot)$ is continuously differentiable on the positive domain, strictly increasing, strictly concave, $u_c(0, s) = \infty$, $u_c(\infty, s) = 0$, $u_s(c, 0) = \infty$, $u_s(c, \infty) = 0$, $u_{cs}(\cdot) \geq 0$, $f(\cdot)$ is continuously differentiable, strictly concave, there is no production without inputs, $f(0, 0) = 0$, capital productivity is unbounded if the capital stock decreases to zero, $f_k(0, e) = \infty$, and capital maintenance costs exceed capital productivity for large capital stocks, $f_k(\infty, e) = -\delta < 0$. Moreover, the exhaustible resource is non-essential for production, $0 < f(k, 0) < \infty$ for $k > 0$, the marginal productivity of resource extraction, $f_e(k, e)$, is bounded from above, $0 < f_e(k, 0) < \infty$. These assumptions on the production function ensure that the resource price does not increase without bound if extraction becomes zero. Next, the regeneration function $g(\cdot)$ is taken to be continuously differentiable; the resource cannot recover if completely exhausted, $g(0) \leq 0$, $0 \leq g'(s) < \infty$, and there is an upper bound on the resource stock that the environment can support in the long run: $\exists \bar{s} > 0 : g(\bar{s}) = 0$, and $g(s) < 0$ if $s > \bar{s}$, and long-run equilibria must lie on the interval $[0, \bar{s}]$.¹ Finally, the resource is called exhaustible if there is no regeneration possible, i.e. if $g(\cdot) \equiv 0$.

¹ For example, the maximal forest area is bounded by the total land area.

It follows from the strictly positive maintenance costs of capital, $f_k(\infty, e) = -\delta < 0$, that the set of attainable states for (k, s) is bounded. Thus, we can restrict the state space to a compact sub-set of \mathbf{R}_+^2 , that can be depicted graphically.

The properties of the utility functions and the production function ensure that in the optimum, consumption c , man-made capital k , and the resource level s are strictly positive, $c > 0, k > 0, s > 0$, and the first-order conditions for the program in current prices include

$$p = u_c(c, s), \quad (4)$$

$$q \geq p f_e(k, e) \perp 0 \leq e, \quad (5)$$

$$\dot{p} = \rho p - p f_k(k, e), \quad (6)$$

$$\dot{q} = \rho q - u_s(c, s) - g'(s)q, \quad (7)$$

where the complementarity \perp denotes that $p f_e(\cdot) = q$ is required for $e > 0$, while inequality may hold if $e = 0$. Let us normalize prices so that the price for the consumer good is unity. That is, we divide all prices by p , and define $r = \rho - \dot{p}/p$ as the rate of price depreciation. The first-order conditions (4)-(7) are now rewritten:

$$r = \rho - (u_{cc}(c, s)/u_c(c, s)) \dot{c}, \quad (8)$$

$$q \geq f_e(k, e) \perp 0 \leq e, \quad (9)$$

$$r = f_k(k, e), \quad (10)$$

$$\dot{q} = -u_s(c, s)/u_c(c, s) + (r - g'(s))q. \quad (11)$$

Together with the constraints (2) and (3), which are binding if dual variables are positive, conditions (8)-(11) determine the optimum of the convex welfare program (1)-(3). The steady state equilibrium conditions follow for \dot{k} , \dot{c} , and \dot{q} equal to zero, and $r = \rho$:

$$c = f(k, e), \quad (12)$$

$$e = g(s), \quad (13)$$

for the program constraints (2) and (3), and

$$f_e(k, e) \leq q \perp 0 \leq e, \quad (14)$$

$$\rho = f_k(k, e), \quad (15)$$

$$(\rho - g'(s))q = u_s(c, s)/u_c(c, s), \quad (16)$$

for the first-order conditions (8)-(11). The number of equations precisely matches the (number of) state variables (c, e, k, s, q) , implying that the set of solutions consists of distinct points, provided that the Jacobian of the equations is regular at all solutions. This is commonly assumed to be the case almost everywhere, because singularity of the Jacobian is considered exceptional (Sard's theorem). However, we now show that eliminating all resource regeneration introduces a structural singularity that causes the steady states to form a continuum.

3. State dynamics for an exhaustible resource

To prepare for the analysis of the consequences of limited regeneration in the next section, we write $\alpha g(\cdot)$ for the regeneration function and in this section assume exhaustibility by setting $\alpha = 0$. Exhaustibility implies that extraction must be zero in the steady state ($e = 0$), and the steady state conditions (12)-(16) reduce to:

$$c = f(k, 0), \quad (17)$$

$$e = 0, \quad (18)$$

$$f_e(k, 0) \leq q, \quad (19)$$

$$\rho = f_k(k, 0), \quad (20)$$

$$\rho q = u_s(c, s)/u_c(c, s). \quad (21)$$

For reference, we identify the steady state by its state variables k and s , since these determine directly the flow variables c and e from (17) and (18), while the supporting price q is calculated from (21). A steady state is said to be ‘maximal’ if equation (19) is binding, and this will be denoted by hats.

The state-space analysis proceeds in three steps. First, we characterize the maximal steady state by showing that it exists and is unique, that it is the largest element of the continuum of steady states, and that it yields maximal welfare. Secondly, we describe the two possible state flow diagrams associated with this model. In a two-dimensional plane, a first-order differential equation whose eigenvalues have negative real parts converges to either a node (if eigenvalues are real-valued) or a spiral sink (if eigenvalues are complex) [Hirsch and Smale 9, section 5.4]; see the annex for a graphical illustration. The distinction matters because unlike a node, a spiral sink can lead to a steady state whose resource stock is less than maximal even if the economy starts above maximal level. Thirdly, we show that if the consumer’s discount factor is chosen sufficiently small, resource extraction is ruled out. This illustrates that in this model besides the date of implementation of efficient policies, the present generation’s care for the welfare of future generation also has lasting effects.

PROPOSITION 1. There exists a maximal steady state (\hat{k}, \hat{s}) , which is positive and unique.

Proof. Choose \hat{k} such that $\rho = f_k(\hat{k}, 0)$; this is possible because $f(\cdot)$ is continuously differentiable, $f_k(0, 0) = \infty$, and $f_k(\infty, 0) \leq 0$, and the resulting value will be positive. Moreover, the stock \hat{k} is unique due to strict concavity of $f(\cdot)$. Furthermore, calculate the positive numbers $\hat{c} = f(\hat{k}, 0)$, $\hat{q} = f_e(\hat{k}, 0)$; finally, find a value \hat{s} such that $u_s(\hat{c}, \hat{s}) = \rho \hat{q} u_c(\hat{c}, \hat{s})$. This value is unique and positive since $u(\cdot)$ is continuously differentiable, strictly concave, $u_c(\hat{c}, \hat{s})$ is increasing in \hat{s} , $u_s(\hat{c}, \infty) = 0$, and $u_s(\hat{c}, 0) = \infty$. ■

The next proposition defines the continuum of steady states and shows that the maximal steady state has indeed the maximal level of environmental resource:

PROPOSITION 2. *The steady states form a continuum, defined by the set $\{(k, s) \mid k = \hat{k}, 0 < s \leq \hat{s}\}$.*

Proof. We start by showing that every element of $\{(k, s) \mid k = \hat{k}, 0 < s \leq \hat{s}\}$ represents a steady state. Choose $\hat{c} = f(\hat{k}, 0) > 0$ as above, $e = 0$, so that (17), (18) and (20) are satisfied. Furthermore, for any resource level satisfying $0 < s \leq \hat{s}$, the second derivatives $u_{ss}(\cdot) < 0$ and $u_{cs}(\cdot) \geq 0$ ensure that $q = u_s(\hat{c}, s) / (\rho u_c(\hat{c}, s)) \geq \hat{q} = f_e(\hat{k}, 0)$; hence, (19) is also satisfied. Conversely, equations (17), (18) and (20) uniquely determine k , c , and e . No resource level such that $s > \hat{s}$ leads to a value of q in (21) that will also satisfy (19). ■

Consumption being equal among steady states, it is trivial to show that the maximal steady state has maximal welfare.

PROPOSITION 3. *Of all steady states, welfare is maximal in (\hat{k}, \hat{s}) .*

Proof. Evident, since the utility function is increasing in s . ■

We are now ready to describe the state flow diagram of this model. Figure 1 refers to the case that the maximal steady state, denoted by B, is a node. The continuum of steady states is represented by BD, and arrows denote the direction of motion of the state variables. The dashed line ABC denotes the border to the right of which extraction is zero ($e = 0$), and to the left of which extraction is strictly positive ($e > 0$). The optimal paths EB and FB mark the border above which the allocations converge to the maximal steady state B, and below which the allocations converge to a sub-maximal steady state on BD. Within the basin below EBF, different initial states can lead to the selection of different steady states. Consequently, in the upper basin the effect of postponing efficient policies vanishes eventually, as in the original Ramsey model, whereas in the lower basin the effect persists.

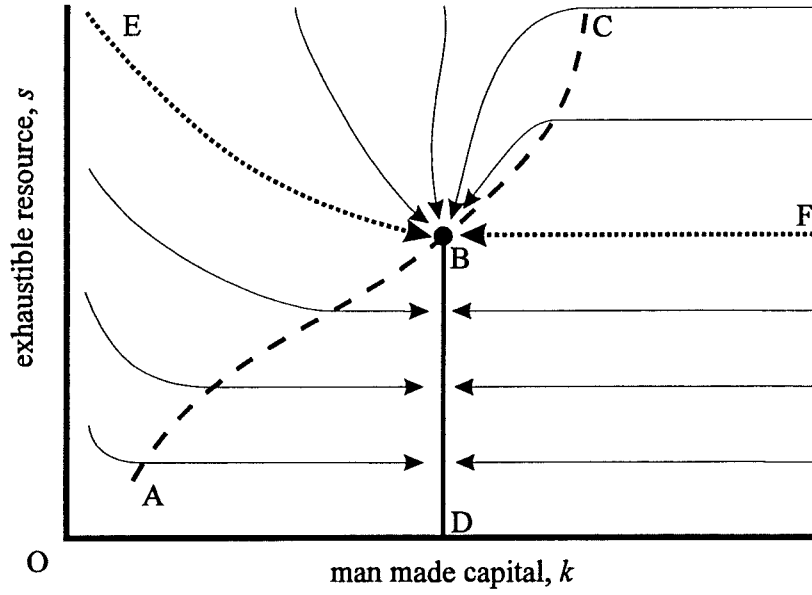


Figure 1. Steady state selection in case of a node

This figure expresses various qualitative properties of the state dynamics of the model. To indicate how these properties follow from the assumptions made, we list them separately as Propositions 4-10, for which proofs are given in the annex. Let us start with global stability.

PROPOSITION 4. *For any initial allocation (k_0, s_0) , there is a unique optimal path that converges to a steady state on BD.*

It follows immediately from Proposition 4 that flow variables (c_0, e_0) are a function of (k_0, s_0) , and thus, more generally, that we can write (c_t, e_t) as a function of (k_t, s_t) and this enables us to restrict our analysis to the dynamics of (k_t, s_t) . For a resource level s below the maximal steady state \hat{s} , the dynamics around the continuum BD satisfy:

PROPOSITION 5. *For all steady states except the maximal steady state, there is a neighborhood in which resource extraction is zero along every optimal path.*

Hence, sub-maximal steady states can only be reached via an allocation path with zero extraction from a certain period onwards, or $e_t = 0$ for $t \in [T, \infty)$ for some T , and in Figure 1 all paths converging to BD must eventually become horizontal. Paths on the right-hand side of BD have the following property.

PROPOSITION 6. *If $s_0 \leq \hat{s}$, and $k_0 \geq \hat{k}$, then the optimal solution follows a trajectory with zero extraction of the resource and with decreasing man-made capital.*

This proposition implies that if, initially, (i) the exhaustible resource is below the maximal steady state level, and (ii) the renewable resource exceeds the steady state level, then the economy will temporarily increase its consumption by not fully replacing the man-made capital stock, and will abstain from extraction of the exhaustible resource. Next, we draw the line between the two regions where $e > 0$ and $e = 0$, respectively.

PROPOSITION 7. *The line ABC in Figure 1 that marks the boundary between positive and zero extraction along the optimal paths is upward sloping around the maximal steady state B and has no kink at B.*

We can now characterize the border EBF that separates in Figure 1 the basin of the steady state at point B from the continuum BD.

PROPOSITION 8. *In case of a proper or an improper node, there is a line EBF in Figure 1 that marks the boundary between allocations that converge to the maximal steady state (above the line), and allocations that converge to a sub-maximal steady state (below the line). Optimal paths do not cross EBF. The line EB is downward sloping, EBF has a kink at B, and BF is horizontal.*

The previous proposition completes our analysis of Figure 1 and model (1)-(3) in case the steady state is a node. We now turn to the case of a spiral sink, depicted in Figure 2.

PROPOSITION 9. *If the maximal steady state is a spiral sink, then there is a line BF as in Figure 2, for which all allocations converge to the maximal steady state. All other initial allocations converge to a sub-maximal steady state. Optimal paths do not cross BF. The line BF is horizontal.*

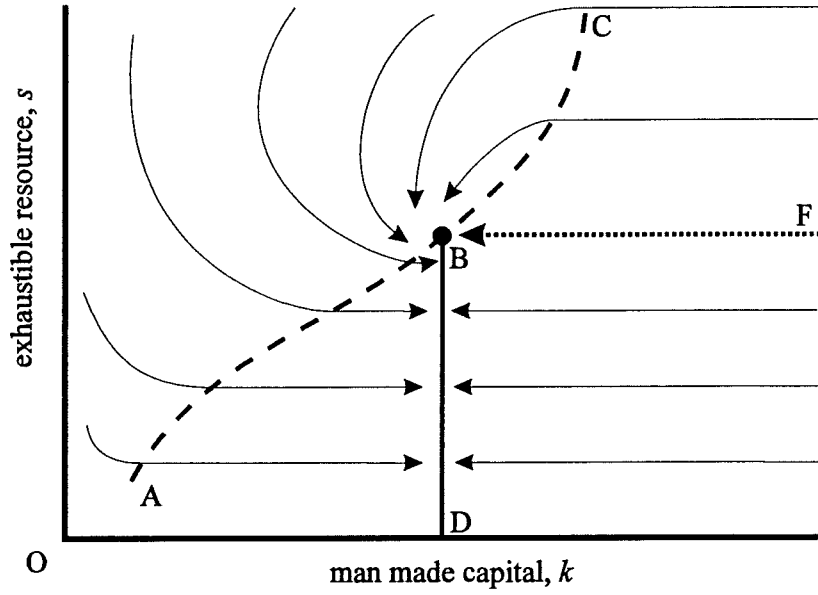


Figure 2. Steady state selection in case of a spiral sink

As can be seen in the figure, all paths starting above the maximal steady state will eventually cross the line AB, and end up at a sub-maximal steady state. This reflects the property that if the planner is impatient he will squander resources at the expense of future generations. However, this property should be interpreted with care because the point B itself depends on the rate of time preference. It shifts to the upper right as the planner becomes more patient (see proof of Proposition 1) and, for given initial stocks makes it more likely for the initial state to be at the quadrant left of BD. Since the paths of the node and spiral sink are qualitatively the same at this quadrant, for low values of the time preference ρ , the distinction between the node and spiral sink becomes insignificant. In this quadrant, the optimal paths generally show an initial reduction in the resource stock, but this reduction is itself a representation of impatience. Indeed, resource extraction decreases if the time preference ρ decreases, and for sufficiently small discount rate ρ , there will be no resource extraction at all.

PROPOSITION 10. *For any initial allocation (k_0, s_0) , for the discount rate ρ sufficiently close to zero, extraction will be zero from $t = 0$ onwards.*

4. State dynamics for a resource with limited regeneration

We consider the case in which the resource has a small, but nonnegative regeneration capacity. For this, we explicitly introduce the scaling factor α , into (12)-(16), which changes these equation into

$$c = f(k, e), \quad (22)$$

$$e = \alpha g(s), \quad (23)$$

for the program constraints and

$$f_e(k, e) \leq q \perp e \geq 0, \quad (24)$$

$$\rho = f_k(k, e), \quad (25)$$

$$(\rho - \alpha g'(s))q = u_s(c, s)/u_c(c, s), \quad (26)$$

for the first-order conditions. We have, by definition, that for $\alpha = 0$ both inequalities of (24) are binding at the maximal steady state (\hat{k}, \hat{s}) . Now, if $g(\hat{s}) \geq 0$, that is $\hat{s} < \bar{s}$ where \bar{s} is the maximal stock the environment can support, i.e. such that $g(\bar{s}) = 0$, and if the Jacobian of the set of equations is regular – which is almost everywhere the case – the steady state equations can be solved for small positive variations in α (Implicit Function theorem). Thus, near the maximal steady state of the exhaustible resource economy there exists a distinct steady state of the economy with limited regeneration. The following proposition establishes uniqueness and saddlepoint instability of this steady state, and describes the path in the upper and lower basin.

PROPOSITION 11. *If the resource has limited regeneration (small $\alpha > 0$), and $g(\hat{s}) \geq 0$ where \hat{s} is the maximal steady state of the nearby economy without regeneration ($\alpha = 0$), then the allocations converge to a unique steady state and inherit the qualitative properties described in Proposition 7, except that the line BD now acts as a turnpike to B.*

The proposition states that the line AC separating the domains with $e_t > 0$ from $e_t = 0$ persists as limited resource regeneration is being introduced. The proof in the Annex shows that for $\alpha > 0$ there is a unique steady state close to B for $\alpha = 0$ (close in the sense of, continuously varying in α). The proof further shows that the line AC shifts downward. Below the line AC, extraction is zero and man-made capital dynamics are given by the restricted welfare program, whereas the resource dynamics follow from the now autonomous regeneration process, $\dot{s} = \alpha g(s)$. In the neighborhood of the line BD, the paths exhibit turnpike behavior. For the allocations along the former continuum of steady states BD, the man-made stock remains constant, and the resource stock increases to the steady state level. Other trajectories bend off when they come close to BD to converge to the steady state. This is reflected in Figure 3 for the case of a proper node. Similarly, for the diagrams of the other cases (improper node and spiral sink) the respective earlier dynamics are inherited, except that BD now acts as a turnpike. One obvious limitation of this result is that it does not indicate in quantitative terms how large the factor α can become for the turnpike to be preserved. Yet it is worth to note that as soon as α is positive, the technology would in principle allow the economy to return to its maximal sustainable level wherever it is, since it has infinite time to do so.

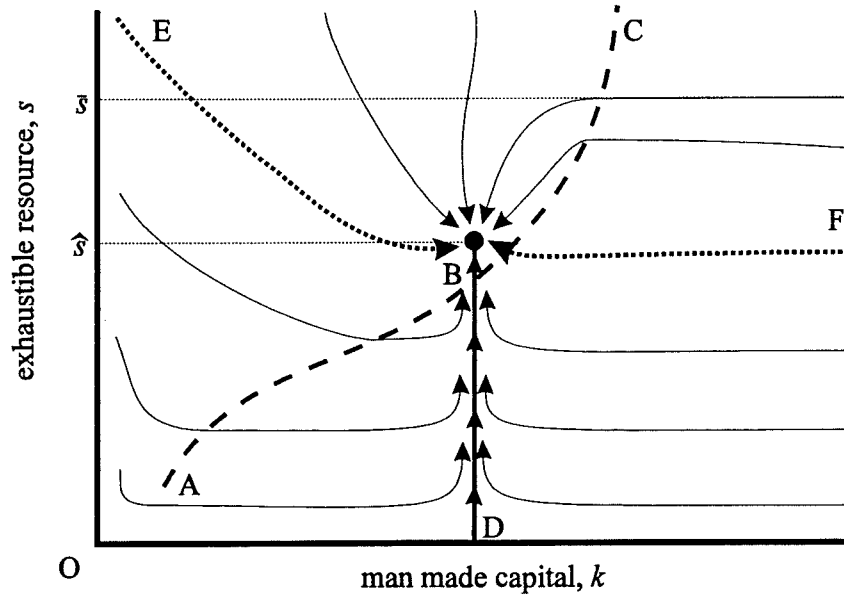


Figure 3. Steady state selection in case of a proper node and regeneration and $\hat{s} < \bar{s}$

The figure illustrates that limited resource regeneration causes manmade capital to converge faster than the resource. Thus, while the effect of postponing the efficient resource policies fades out eventually, as in the original Ramsey model, almost until the very end of time the path stays close to the one without regeneration, and the effect persists.

Finally, let us consider the effect of a reduction in discount rate ρ , which is related to the case that $g(\hat{s}) < 0$ and the assumption of Proposition 11 no longer applies. If the discount rate ρ decreases, the maximal steady state increases (see proof of Proposition 1), leading eventually to $g(\hat{s}) < 0$. Notice that in this case the steady state equations can only be solved for small positive variations in α if $g(s)=0$ (since $e \geq 0$ rules out $g(s) < 0$ and the complementarity (24) rules out $g(s) > 0$). Therefore, the unique steady state for the economy with small regeneration is given by (\hat{k}, \bar{s}) , where \bar{s} is the maximal stock the environment can support, i.e. such that $g(\bar{s}) = 0$. The maximal steady state (of the economy with $\alpha=0$) has now become unattainable, $s_0 \leq \bar{s} < \hat{s}$, and most of the attainable state space lies below the line AC. Thus, for low discount rates, resource extraction is zero for the indefinite future from some T onwards when the economy crosses the line AC. The state dynamics are shown in Figure 4.

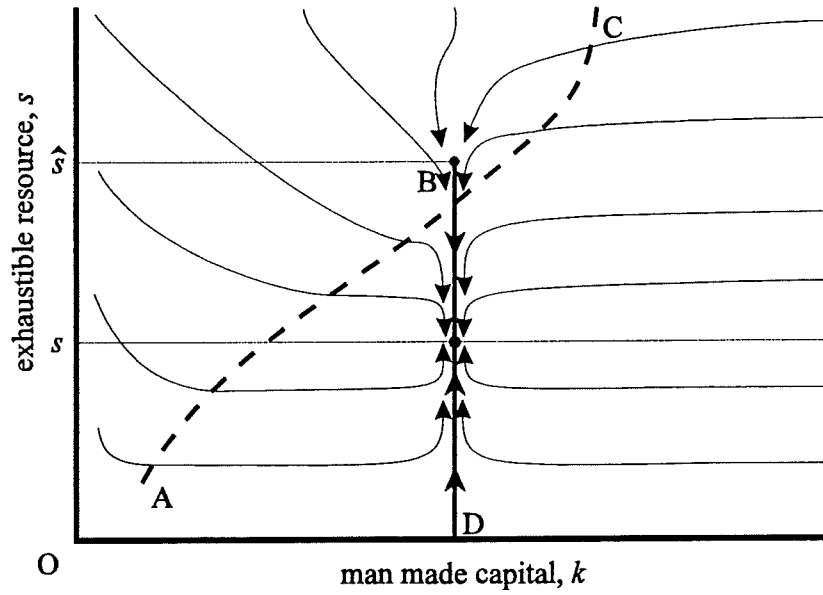


Figure 4. Steady state selection in case of regeneration and $\bar{s} < \hat{s}$

5. Scope for generalisation

The model presented so far only had two stocks and one consumer good, and this raises the question as to which part of the result carries through in a more general setting.

As mentioned in the introduction, the property that the steady state level of the exhaustible resource forms a continuum is robust, although it may happen that no steady state exists, as in Krautkraemer's [11] economy where the extraction value increases without bound. The turnpike property is robust as well, since it only relies on a structural stability argument. It would hold around every continuum of stable steady states. Uniqueness of the optimal trajectory depends on the convexity of the problem and not on its dimension. By the same token, it is easy to extend the number of consumers by allowing for multiple dynastic agents, since we can write the equilibrium as a Negishi welfare program with period-specific utility function $u(c, s) = \max_{c_i \geq 0} (\sum_i \alpha_i u_i(c_i, s) \text{ s.t. } \sum_i c_i \leq c)$, where consumption c_i is taken to be rival, and the amenity value non-rival [Lucas and Stokey 13]. In addition, following the Negishi approach, welfare weights can be adjusted so as to meet the budget constraint of every agent [Ginsburgh and Keyzer 8, Ch.8]².

The next proposition shows that convergence is even preserved under an arbitrary increase in the number of exhaustible resources, also if these enter the production and utility function in a non-convex fashion.

² The welfare weights depend on the distribution of endowments but do not vary over time so that they can be considered fixed for the dynamic program.

PROPOSITION 12. *Let the natural resource stock s and the extraction e form n -dimensional vectors, and assume that the utility and production function satisfy with respect to c and k all other assumptions of section 2 and that resources do not regenerate. Then, the economy converges to a steady state.*

The proposition does not imply global convergence to a unique steady state. We cannot rule out indeterminacy and bifurcations of the optimal path, let alone non-uniqueness of the steady state, and yet, every optimal path will converge. The proposition indicates that for exhaustible resources virtually any technology will do, as long as man made capital is a scalar and satisfies the regular assumptions. We can actually go a step further. As the proof in the annex treats extraction e and resource stock s as mere parameters, it would be possible to let these parameters enter all functions in the model. Utility could be made to depend on extraction and production on stock level.

However, this has taken us far afield, and to reestablish a connection with prevailing concerns of environmental management, it may be helpful to track our way back, and make a stepwise return to the assumptions of the earlier model. We state properties that can be established as we strengthen our assumptions, but for brevity only sketch the line of proof.

(a) *If in addition to the assumptions of Proposition 12, the utility and production function are differentiable, then steady states form continuums* (proof: the structural singularity due to condition (18) in steady state (17)-(21) is the same as for the model with a single resource).

(b) *If in addition to conditions (a), the utility function is increasing in resource stocks, then for a sufficiently small discount rate, resource extraction will be zero from $t=0$ onwards* (proof: as for proposition 10).

(c) *If in addition to conditions (a), the utility function is strictly concave in (c,s) and the production function is strictly concave in (k,e) , and if there is limited resource regeneration, then the continuum serves as turnpike, and the optimum is decentralizable as a competitive equilibrium* (proof: the optimal path is unique, hence it is structurally stable under any perturbation in the neighborhood of the steady state to which the path converges; limited regeneration creates such a perturbation).

(d) *If in addition to conditions (c), the utility function is additively separable, $u(c,s)=v(c)+w(s)$, and there is a maximal stock \bar{s} which the environment can support, i.e. such that $g(\bar{s})=0$, or the rate of discount ρ is sufficiently small, then the steady state is unique and the path is globally convergent* (proof for separable utility: as in the proof of Proposition 1, we search for \hat{s} that solves $u_s(\hat{c},s)=\rho\hat{q}u_c(\hat{c},s)$, given fixed \hat{c} . Under separability, the term on the right-hand side is invariant in s and by strict concavity the Jacobian of the left-hand side term is non-singular for all s ; thus there exists a unique solution; proof for small but positive ρ : as in Figure 4, the economy converges to a unique steady state (\hat{k},\bar{s})).

(e) *If in addition to conditions (d), there is only one resource, we can make the further assumptions on slope and curvature of Section 2 and produce the associated diagrams.*

Finally, it may be noticed that Proposition 12 and properties (a)-(c) extend to OLG economies and economies with discrete time periods cf. [Gerlagh 7, Theorem 2.3].

6. Conclusion

To analyze the long-term effect of postponing the implementation of efficient resource management practices, we have described the state dynamics of a Ramsey model that was extended by inclusion of a renewable resource with amenity value and limited regeneration capacity. Despite the extension the uniqueness and saddlepoint stability of the optimal trajectory of the original model could be maintained.

In the absence of regeneration, the resource becomes exhaustible and steady states form a continuum consisting of a line segment with constant man-made capital and a level of the resource between zero and a maximal value. This level depends on model parameters, such as the rate of discount, but is not sensitive to changes in initial stock levels. The optimal trajectory selects one equilibrium solution from this continuum, on the basis of these initial stock levels. Hence, there is path dependence and postponement has lasting effects. Indeed, in the absence of regeneration, it is obvious that an initial stock of the resource below the maximal value cannot return to that value, but the question analyzed is how much deeper it will fall. Furthermore, even when the initial resource level exceeds the maximal value, the economy could, if the consumer's rate of discount is too high, end up in a sub-maximal steady state. If the resource has a very limited as opposed to a zero rate of regeneration, the qualitative properties of the trajectories are preserved, except that the path, rather than ending at some point on the continuum, eventually bends off, as on a turnpike, to move, alongside the continuum, towards the steady state. Finally, many of the qualitative properties extend to larger dimensions, for example to the case with multiple resources that cannot regenerate. Yet two main limitations of our results are that man-made capital was assumed to be a scalar, and that we did not provide any quantitative measure of how large the regeneration capacity can become for the turnpike property to be preserved.

Annex. Proofs of propositions 4-12

PROOF OF PROPOSITION 4. *For any initial allocation (k_0, s_0) , there is a unique optimal path that converges to a steady state on BD.*

Proof. First, we prove uniqueness of an optimal solution of (1)-(3) by an indirect demonstration. Because of strict concavity of $u(\cdot)$, if there are two solutions with equal welfare, the solution in between is also feasible and returns strictly higher welfare. Second, we prove global convergence. It follows from uniqueness of the optimal path that that we can write (c_t, e_t) as a function of (k_t, s_t) , and thus $(\dot{k}, \dot{s}) = \Gamma(k, s)$ for some operator Γ . Thus, optimal paths in the (k, s) -space do not cross. Because of compactness of the state space,

any path converges to the non-empty limit-set, which in a two-dimensional plane, by the Poincaré-Bendixson theorem, is consisting of steady state points and closed orbits (cycles). Non-increasingness of s rules out cycling. Thus, the limit-set consists of steady states only. Since the continuum BD contains all steady states (Proposition 2), all paths converge to it. ■

PROOF OF PROPOSITION 5. *For all steady states except the maximal steady state, there is a neighborhood in which resource extraction is zero along every optimal path.*

Proof. We analyze the dynamics of a restricted welfare program that includes an explicit zero extraction condition, $e=0$, in the welfare program (1)-(3). This amounts to replacing the first order condition (5) by $e=0$. The model reduces to a standard Ramsey one-capital stock model which stock and flow dynamics are given by:

$$\dot{c} = (\rho - f_k(k,0)) u_c(c,s)/u_{cc}(c,s) s, \quad (27)$$

$$\dot{k} = f(k,0) - c, \quad (28)$$

for fixed s . Now the usual phase diagram applies cf. [Blanchard and Fischer 1, Ch.2, Fig. 2.3]. As can be seen in such a phase diagram, along the optimal path, consumption is a strictly increasing function of the man-made capital stock, $c = \zeta(k)$ with $\zeta'(k) > 0$.

The dynamics of the unrestricted welfare program in Figure 1 coincide with those of the restricted welfare program if the unrestricted optimal path has no resource extraction, that is, if constraint (5) holds for the unrestricted program. Proving this amounts to showing that $pf_e(.) \leq q$ for the restricted optimal paths.

Because for an optimal restricted path with fixed resource level s , the optimal consumption is a continuous increasing function of man-made capital $\zeta(k)$, the relative price for the resource level, $q = u_s(c,s)/(\rho u_c(c,s))$, is also a continuous function of k . If the initial resource level s lies below the maximal steady state level, it follows that this price q exceeds the (continuous) marginal productivity of the resource, $f_e(.)$, in the steady state (see proof of Proposition 2). Because of continuous differentiability, $f_e(.) < q$ in a neighborhood, and the restricted allocation path satisfies (5); it is thus also a solution of the unrestricted program. ■

PROOF OF PROPOSITION 6. *If $s_0 \leq \hat{s}$, and $k_0 \geq \hat{k}$, then the optimal solution follows a trajectory with zero extraction of the resource and with decreasing man-made capital.*

Proof. We solve the restricted program with $e=0$ from the proof of the previous proposition, and it follows directly from the same arguments that on a path where $k_0 \geq \hat{k}$, all first order conditions (4)-(7) are satisfied. ■

PROOF OF PROPOSITION 7. *The line ABC in Figure 1 that marks the boundary between positive and zero extraction along the optimal paths is upward sloping around the maximal steady state B and has no kink at B.*

Proof. Because of concavity of the utility and production function, the first order conditions (8)-(11) are necessary and sufficient, and the flow function $\Gamma: (k, s) \rightarrow (\dot{k}, \dot{s})$ is continuous and differentiable for the interior domains of $e > 0$ and $e = 0$. Locally around the maximal steady state B, we can thus write:

$$(\dot{k}, \dot{s})' = D\Gamma(\tilde{k}, \tilde{s})', \quad (29)$$

where D is the differential operator, $D\Gamma$ is a matrix, and tildes denote deviations from the maximal steady state B. The local (linear) equation holds everywhere except for the boundary between the domains of $e > 0$ and $e = 0$, that is, except for the line ABC. Now, let us write $\Gamma_{e>0}$ for the differential operator of the paths where $e > 0$, and accordingly $D\Gamma_{e=0}$ for the linearization of the paths where $e = 0$. Because of global convergence, $D\Gamma_{e>0}$ is either a node or a spiral sink. The line ABC is locally spanned by $(D\Gamma_{e>0})^{-1}(1, 0)'$, that is, by those elements for which $\dot{s} = 0$. Hence ABC is locally linear and has no kink at B. It follows from Proposition 6 that the right lower corner DBF has zero extraction and thus lies on one side of the line ABC; hence ABC is upward sloping. We thus have the operator $\Gamma_{e>0}$ for the left-upper side of ABC and $\Gamma_{e=0}$ for the right-down side of ABC. ■

PROOF OF PROPOSITION 8. *In case of a proper or an improper node, there is a line EBF in Figure 1 that marks the boundary between allocations that converge to the maximal steady state (above the line), and allocations that converge to a sub-maximal steady state (below the line). Optimal paths do not cross EBF. The line EB is downward sloping, EBF has a kink at B, and BF is horizontal.*

Proof. The proof proceeds in three stages: (a) Unrestricted local dynamics, (b) Restricted local dynamics, (c) Restricted global dynamics.

(a) *Unrestricted local dynamics*

To characterize the global dynamics in the restricted case we must describe local unrestricted dynamics in more detail, that is, we analyze the operator $\Gamma_{e>0}$. First, notice that the state dynamics around the steady state (29) are point symmetric: if two initial states 'a' and 'b' are equal up to a multiplication factor λ :

$$\begin{pmatrix} \tilde{k}_0^a \\ \tilde{s}_0^a \end{pmatrix} = \lambda \begin{pmatrix} \tilde{k}_0^b \\ \tilde{s}_0^b \end{pmatrix}, \quad (30)$$

then this will be valid for all $t > 0$:

$$\begin{pmatrix} \tilde{k}_t^a \\ \tilde{s}_t^a \end{pmatrix} = \lambda \begin{pmatrix} \tilde{k}_t^b \\ \tilde{s}_t^b \end{pmatrix}. \quad (31)$$

Secondly, we characterize the convergence properties by graphic display. Hirsch and Smale [9, Section 5.4] show that in the two dimensional plane locally stable points fall into three categories: proper nodes (if $D\Gamma_{e>0}$ has two distinct real eigenvalues), improper nodes (real, non-distinct eigenvalues) and spiral sinks (complex eigenvalues). Nodes with real values are “almost surely” proper (Hirsch and Smale, 1974, Ch.7, Theorem 1) when obtained from the Jacobian of a nonlinear function, but we discuss the improper node for completeness and because it can be viewed as a transition between the other two cases.

For a proper node, the flow dynamics around the maximal steady state look like Figure 5.

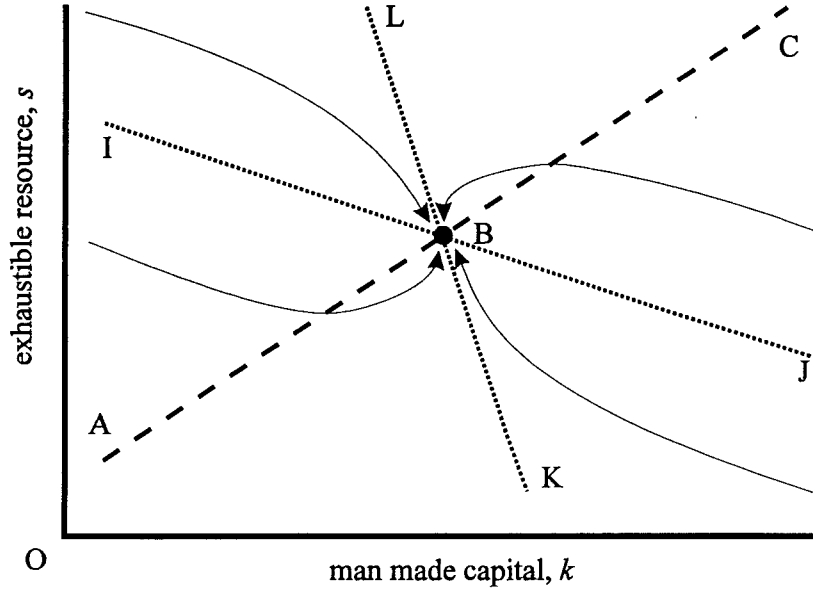


Figure 5. Unrestricted state dynamics around a proper node

where the lines IJ and KL denote the eigenspaces with the relatively largest and smallest negative eigenvalues, respectively, and AC denotes the line with zero extraction, which is spanned by $(D\Gamma_{e>0})^{-1}(1,0)'$.

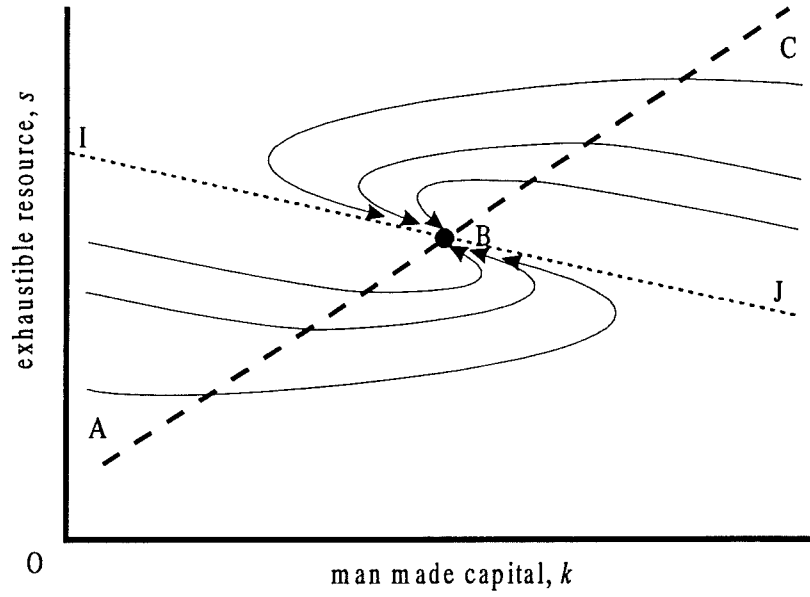


Figure 6. Unrestricted dynamics around an improper node

Figure 6 depicts an improper node (rare event) and Figure 7 a spiral sink (not a rare event). The improper node is a special case of the proper one: the two eigenvalues become equal, the angles JBI and KBL become zero, and IJ coincides with KL. The spiral sink corresponds to complex eigenvalues and is therefore not a special case of the node.

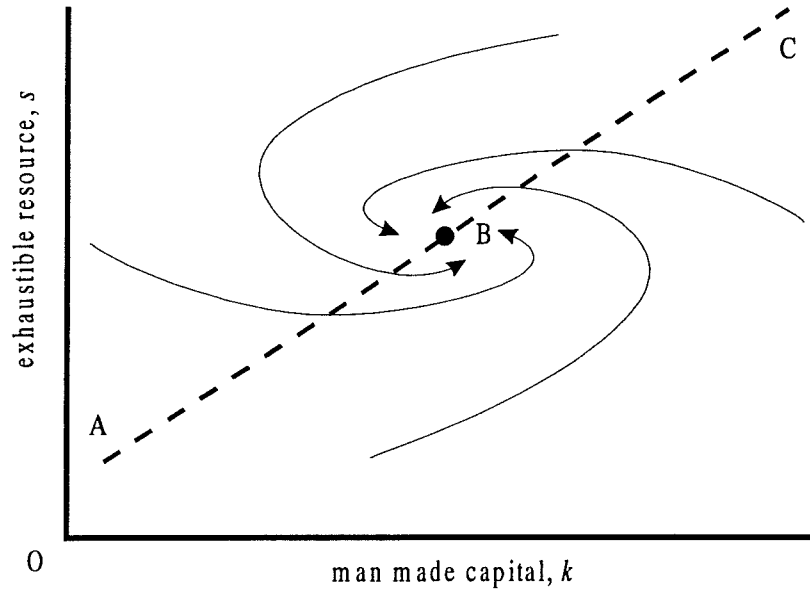


Figure 7. Unrestricted dynamics around a spiral sink

The transition between the three cases is as follows. The eigenvalues are the two roots of the quadratic characteristic equation. As the complex part becomes smaller, the two roots come closer and the spiral sink turns into an improper node, which becomes proper as the real parts start diverging.

(b) *Restricted local dynamics*

So far, we only discussed convergence of the unrestricted path but we already included the line AC of zero extraction. Since the matrix $D\Gamma$ is regular, this line AC exists, and because of Proposition 6, it must be upward sloping. To the left of AC, extraction is positive ($e > 0$), and for this part, local Figure 5 and Figure 6 agree with the global Figure 1. However, to the right of this line AC, the optimal paths cannot follow the unrestricted paths depicted in the local figures because extraction is bounded from below by $e \geq 0$. Consequently, in this part the dynamics follow model (27)-(28).

Figure 8 combines both types of dynamics locally around a proper node:

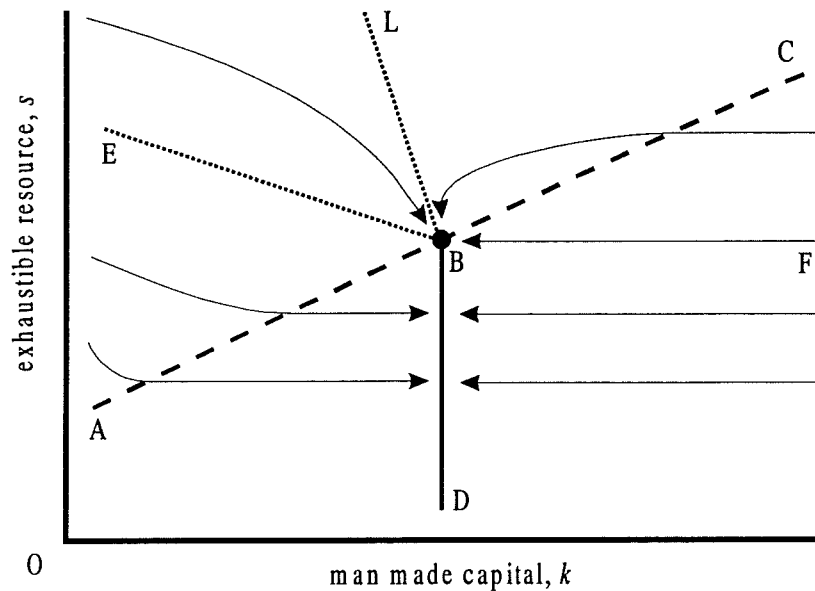


Figure 8. Restricted state dynamics around a proper node

which is the “close up” for the maximal steady state B in Figure 1, that shows the global dynamics. For an improper node, Figure 6, the angle EBL becomes zero.

(c) *Restricted global dynamics*

We are now ready to characterize the set of initial values in the neighborhood of the steady state that converge to the maximal steady state. In the figures for the restricted dynamics, we substituted E for I, where I denotes the eigenspace for the linearized dynamics system, and E denotes the basin border in Figure 1 for the non-linear dynamic system. We show that E “agrees” with I.

Because of the point symmetry around B of (29), any path emanating from an initial state within the triangle IBA in Figure 8 will cross the line AB and converge to a sub-maximal steady state on BD. Similarly, any path emanating from an initial state within the triangle CBF will cross the line BC and converge to the maximal steady state. Hence, every initial state above the line IBF follows an optimal path that remains above IBF and finally converges to the maximal steady state B. All other initial states converge to a sub-maximal steady state. The line IB cannot be upwards sloping because by definition it is on that side of the line ABC where state dynamics are downward sloping. It must therefore be downwards sloping itself. The line BF is horizontal by Proposition 6, hence, the line IBF has a kink at B. The Point E in Figure 1 can be found by solving the differential equation of Γ backwards, starting from any point on IB, in other words, E is in line with I. ■

PROOF OF PROPOSITION 9. *If the maximal steady state is a spiral sink, then there is a line BF as in Figure 2, for which all allocations converge to the maximal steady state. All other initial allocations converge to a sub-maximal steady state. Optimal paths do not cross BF. The line BF is horizontal.*

Proof. It follows from Proposition 6 that all allocations on BF converge to the maximal steady state, that BF is horizontal, and that all allocations to the right of BD and below BF converge to a sub-maximal steady state. Furthermore, we prove that all other allocations converge to a sub-maximal steady state as well. In case of a spiral sink, all initial allocations to the left of ABC eventually cross the line ABC when they spiral towards the steady state (Figure 7). Thus, any path emanating from an initial state to the left of ABC in Figure 2 will cross the line AB and converge to a sub-maximal steady state on BD. Furthermore, all allocations to the right of BC and above BF cross the line BC and enter the area to the left of ABC, also converging to a sub-maximal steady state on BD. ■

PROOF OF PROPOSITION 10. *For any initial allocation (k_0, s_0) , for the discount rate ρ sufficiently close to zero, extraction will be zero from $t = 0$ onwards.*

Proof. The proposition amounts to showing that for sufficiently small $\rho > 0$, extraction is zero in all periods. We turn to the first-order conditions (4)-(7), and conclude from (7) that because $u_s(c_t, s_t)$ is positive for all stock levels, the resource price q can be increased without bound by reducing ρ :

$$q_t \geq e^{-\rho t} u_s(c_t, s_t) / \rho, \quad (32)$$

(recall that $g(\cdot) \equiv 0$), and since c is bounded away from zero, say, by c^{LB} (this follows from convergence to a steady state where c is bounded away from zero), we have from (4) that:

$$p_t \leq e^{-\rho t} u_s(c^{LB}, 0) \quad (33)$$

Finally, since we assumed that $0 < f_e(k,0) < \infty$, it follows that for p sufficiently close to zero:

$$p_t f_e(\cdot) < q, \quad (34)$$

for all t , which gives $e_t = 0$ by condition (5). ■

PROOF OF PROPOSITION 11. *If the resource has limited regeneration (small $\alpha > 0$), and $g(\hat{s}) \geq 0$ where \hat{s} is the maximal steady state of the nearby economy without regeneration ($\alpha = 0$), then the allocations converge to a unique steady state and inherit the qualitative properties described in Proposition 7.*

Proof. First, we prove in two steps that there is one unique steady state; (i.a) we show that any steady state for $\alpha > 0$ must be close to a steady state for $\alpha = 0$, and (i.b) we show that of all the steady states on the continuum for $\alpha = 0$, only the maximal steady state can produce a steady state nearby for $\alpha > 0$. (i.a) Recall Sard's theorem, which states that the Jacobian of (22)-(26) will be regular almost everywhere. Any steady state of the economy with small regeneration must lie close to one without regeneration, because, if there is a solution of (22)-(26) for $\alpha > 0$, then the equations can be solved for a small variation in α , particularly for α becoming zero. The steady state (k, s, c, e, q) is thus an implicit function of α . (i.b) For $\alpha = 0$, the maximal steady state is the unique steady state for which both inequalities of (24) hold. As above, this steady state persists under small variations in α . Other sub-maximal steady states have $f_e(k,0) < q$, but then the steady state equations (22)-(26) cannot be solved for a small increase in α , since $e = \alpha g(s)$ returns a strictly positive extraction level $e > 0$. Therefore, we must have $f_e(k,e) = q$, and this cannot be accomplished by a small variation of k or q . Thus, there is one unique steady state for $\alpha > 0$ close to the maximal steady state.

Second, we prove convergence. Hirsch and Smale [9, Ch.16] show that, whenever the steady states are regular, the steady states and the local paths around them will vary continuously in the parameters (here α). Thus, all paths in the upper basin converging to the maximal steady state in case of no regeneration will continue to do so in case of small but positive regeneration (structural stability).

This leaves us with the analysis of the lower basin, and particularly, with the domain to the right of AC, and the dynamics around the former continuum BD. We return to part (a) of the proof of Proposition 8. We have from (29) that \dot{k} and \dot{s} are linear in k and s , locally around the steady state B where they are zero. We following the argument just below Figure 5 that specified the line AC as being spanned by $D\Gamma_{e>0}^{-1}(1,0)'$. To maintain $e=0$ at AC, we now have that AC is translated by $D\Gamma_{e>0}^{-1}(0, g(\hat{s}))$, since $e = g(\hat{s})$ in the steady state B. As extraction is positive to the left of AC, it must be that B is at the left, and thus, AC is translated right-downwards.

For $e_t = 0$, resource dynamics are given by $\dot{s} = g(s)$, and man-made stock dynamics essentially follow the same path as under no regeneration. Particularly, BD varies continuously in α . For sufficiently low values of α , the resource dynamics are of second

order compared to the man-made stock dynamics, allowing BD to serve as a ‘turnpike’. For points on BD, the man-made stock remains constant, and the resource stock increases to its steady state level. ■

PROOF OF PROPOSITION 12. *Let the natural resource stock s and the extraction e form n -dimensional vectors, and assume that the utility and production function satisfy with respect to c and k all other assumptions of section 2 and that resources do not regenerate. Then, the economy converges to a steady state.*

Proof. We start proving convergence. The boundedness of the constraint set and the positive rate of discount ensure that there exists a (possibly non-unique) optimum path $\{c_t, k_t, e_t, s_t\}$ defined for all $t \in [0, \infty)$. Now along any optimal path the sequence $\{e_t, s_t\}$ always converges to some value $(0, s^*)$, since the sequence $\{s_t\}$ is monotonically decreasing. Now consider the standard Ramsey^(0, s*) equilibrium path $\{c_t, k_t, 0, s^*\}$ for fixed zero extraction and fixed $s_t \equiv s^*$. Under the prevailing assumptions this path converges globally to a unique steady state $(c^*, k^*, 0, s^*)$, and the optimal path varies continuously for small perturbations in $f(\cdot)$ and $u(\cdot)$. Interpreting the sequence $\{e_t, s_t\}$ as varying parameters of $f(\cdot)$ and $u(\cdot)$, it follows that the optimal path of $\{c_t, k_t\}$ for the many-resources economy converges to the optimal path of $\{c_t, k_t\}$ for the Ramsey^(0, s*) economy, and thus, $\{c_t, k_t\}$ converges to (c^*, k^*) in the many resources economy as well. Hence, $\{c_t, k_t, e_t, s_t\}$ converges to $(c^*, k^*, 0, s^*)$. ■

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